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A Counterexample to a Global Optimization Algorithm

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Abstract. A counterexample to an algorithm of Dien (1988) for solving a minimization problem with a quasiconcave objective function and both linear and a reverse-convex constraint shows that this algorithm needn't lead to a solution of the given problem.

Key words: Global optimization, quasiconcavity, reverse-convex contraints.

Let be

$$(P) \quad MIN\{f(x) \mid x \in D_A, g(x) \geq 0\}$$

where $f : R^n \to R$ is a bounded quasiconcave function, $g : R^n \to R$ is a convex function and

$$D_A := \{x \in R^n \mid Ax \leq b\}$$

is a bounded polyhedron containing at least one nondegenerated extreme point. A branch & cut algorithm based on simplex subdivisions was given by Dien (1988) for solving (P). The algorithm can fail if the solution of a surrogate problem for determining simplices – generated in the subdivision process – which can be deleted is not determined uniquely.

Let be

$$f(x_1, x_2) = -x_1^2 - x_2^2,$$
$$D_A := \{x \in R^2 \mid x_1 - x_2 \leq 0,5; 0,5x_1 + x_2 \leq 0,5; 0 \leq x_1 \leq 1; x_2 \geq 0\}$$

and

$$g(x) := x_1^2 - x_1 + x_2^2 - x_2.$$

The solution of (P) is $\bar{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Choosing the nondegenerated extreme point $a^o := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ of D_A the start simplex

$$S^o := \text{conv}\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 2 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right\}$$

containing D_A is constructed by solving the linear programming problem

$$(L) \quad MAX\{e^T Q^{-1} x \mid x \in D_A\}$$

where Q is formed by the adjacent vertices of a^o in D_A.

After the second iteration S^o is reduced to the polyhedron

$$\text{conv}\left\{\begin{pmatrix}0\\0\end{pmatrix};\begin{pmatrix}0\\0,5\end{pmatrix};\begin{pmatrix}1\\1\end{pmatrix};\begin{pmatrix}1,25\\0,75\end{pmatrix};\begin{pmatrix}0,5\\0\end{pmatrix}\right\}.$$

In the third iteration the simplex

$$\text{conv}\left\{\begin{pmatrix}0\\0\end{pmatrix};\begin{pmatrix}0\\0,5\end{pmatrix};\begin{pmatrix}1\\1\end{pmatrix}\right\}$$

is deleted (by the bounding process). Because the minimum of a certain surrogate problem which is to be solved in the bounding process is not determined uniquely it is possible to choose either the simplex

$$S^1 := \text{conv}\left\{\begin{pmatrix}0\\0\end{pmatrix};\begin{pmatrix}1\\1\end{pmatrix};\begin{pmatrix}1,25\\0,75\end{pmatrix}\right\}$$

or the simplex

$$S^2 := \text{conv}\left\{\begin{pmatrix}0\\0\end{pmatrix};\begin{pmatrix}1,25\\0,75\end{pmatrix};\begin{pmatrix}0,5\\0\end{pmatrix}\right\}.$$

for determining the next cut. Selecting S^1 the simplex

$$S^3 := \{x \in R^2 \mid x_1 \leq 1\} \cap S^1$$

is used for further investigations. In the next bounding process S^3 is deleted. Thus, $\bar{x} = \begin{pmatrix}1\\1\end{pmatrix}$ is not contained in the set of points which are to search further. The algorithm has not identified the solution of the original problem (P). Using the test

$$\mu(S) > \gamma_k$$

(the symbols are used as in the papers of Dien (1988)) the algorithm works well. If the minimal point in the surrogate problems for determining γ_k is always uniquely determined the test of Dien can be used for identifying the simplex in question and the algorithm provides the solution of the original.

References

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